

A MATTER OF MATTER AND ANTIMATTER

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Abstract

A discrete quantum gravity model given by a quantum sequential growth process (QSGP) is considered. The QSGP describes the growth of causal sets (causets) one element at a time in discrete steps. It is shown that the set \mathcal{P} of causets can be partitioned into three subsets $\mathcal{P} = (\text{ANT}) \cup (\text{MIX}) \cup (\text{MAT})$ where ANT is the set of pure antimatter causets, MAT the set of pure matter causets and MIX the set of mixed matter-antimatter causets. We observe that there is an asymmetry between ANT and MAT which may explain the matter-antimatter asymmetry of our physical universe. This classification of causets extends to the set of paths Ω in \mathcal{P} to obtain $\Omega = \Omega^{\text{ANT}} \cup \Omega^{\text{MIX}} \cup \Omega^{\text{MAT}}$. We introduce a further classification $\Omega^{\text{MIX}} = \Omega_{\text{M}}^{\text{MIX}} \cup \Omega_{\text{A}}^{\text{MIX}}$ into matter-antimatter parts. Approximate classical probabilities and quantum propensities for these various classifications are considered. Some conjectures and unsolved problems are presented.

1 Introduction

This article is mainly a collection of unsolved problems concerning discrete quantum gravity described by a quantum sequential growth process (QSGP). One reason we cannot solve these problems is that we do not know the

classical coupling constants and quantum dynamics of the process. Even if we knew these constants and dynamics, the problems would undoubtedly still be a challenge. Nevertheless, without this complete knowledge we can make some qualitative observations and prove some results that may be useful for this challenge. We also provide some examples that we use as test cases which indicate directions that solutions might take.

A QSGP describes the growth of causal sets (causets) one element at a time in discrete steps. One of the contributions of this article is to observe that the set \mathcal{P} of causets can be partitioned into three subsets

$$\mathcal{P} = (\text{ANT}) \cup (\text{MIX}) \cup (\text{MAT}) \quad (1.1)$$

where we call ANT the set of pure antimatter causets, MAT the set of pure matter causets and MIX the set of mixed matter-antimatter causets. We next observe that there is an asymmetry between ANT and MAT. This may help to answer one of the most important questions in modern physics. Why is there a large preponderance of matter over antimatter in our physical universe? The asymmetry is most easily seen by considering the set of sequential paths Ω in \mathcal{P} . The elements of Ω correspond to possible universe histories. The classification of causets (1.1) can be extended to Ω to obtain the partition

$$\Omega = \Omega^{\text{ANT}} \cup \Omega^{\text{MIX}} \cup \Omega^{\text{MAT}}$$

It turns out that the classical probabilities and quantum propensities of paths in Ω^{ANT} are considerably different than those in Ω^{MAT} . There are indications that Ω^{MIX} dominates Ω^{ANT} and Ω^{MAT} so it is much more likely that our “actual universe” is in Ω^{MIX} . A further classification $\Omega^{\text{MIX}} = \Omega_{\text{M}}^{\text{MIX}} \cup \Omega_{\text{A}}^{\text{MIX}}$ partitions Ω^{MIX} into matter-antimatter parts. Again, there are indications that the propensity of $\Omega_{\text{M}}^{\text{MIX}}$ is considerably larger than the propensity of $\Omega_{\text{A}}^{\text{MIX}}$. The ratio of these propensities may be related to (or even equal to) the proportion of matter to antimatter in our universe and this may eventually be experimentally testable.

2 Quantum Sequential Growth Processes

Let x be a finite nonempty set. A *partial order* on x is a relation $<$ on x that satisfies:

- (1) $a \not< a$ for all $a \in M$ (*irreflexivity*).

(2) If $a, b, c \in x$ with $a < b$ and $b < c$, then $a < c$ (*transitivity*).

If $<$ is a partial order on x we call $(x, <)$ a *partially ordered set* or (*poset*). We think of x as a finite set of space-time points and $<$ as the causal order. Thus, $a < b$ if b is in the causal future of a . For this reason we also call x a *causal set* or *causet* [1, 7]. An element $a \in x$ is *maximal* if there exists no $b \in x$ with $a < b$. For $a, b \in x$ we say that a is an *ancestor* of b and b is a *successor* of a if $a < b$. We say that a is a *parent* of b and b is a *child* of a if $a < b$ and there is no $c \in x$ such that $a < c < b$. In this work we only consider unlabeled causets and isomorphic causets are assumed to be identical.

Let \mathcal{P}_n be the collection of all causets of cardinality n , $n = 1, 2, \dots$, and let $\mathcal{P} = \cup \mathcal{P}_n$. If $x \in \mathcal{P}_n$, $y \in \mathcal{P}_{n+1}$, then x *produces* y if y is obtained from x by adjoining a single element to x that is maximal in y . We also say that x is a *producer* of y and y is an *offspring* of x . If x produces y we write $x \rightarrow y$. We think of x as “growing” into y where the new element is not in the causal past of any element of y . We denote the set of offspring of x by $x \rightarrow$ and for $A \in \mathcal{P}_n$ we use the notation

$$A \rightarrow = \{y \in \mathcal{P}_{n+1} : x \rightarrow y, x \in A\}$$

A *path* in \mathcal{P} is a string (sequence) $\omega = \omega_1 \omega_2 \dots$ where $\omega_i \in \mathcal{P}_i$ and $\omega_i \rightarrow \omega_{i+1}$, $i = 1, 2, \dots$. An *n-path* in \mathcal{P} is a finite string $\omega_1 \omega_2 \dots \omega_n$ where again $\omega_i \in \mathcal{P}_i$ and $\omega_i \rightarrow \omega_{i+1}$. We denote the set of paths by Ω and the set of n -paths by Ω_n . We think of $\omega \in \Omega$ as a possible universal (together with its history) and $\omega \in \Omega_n$ as a possible universe truncated at step n . The set of paths with initial n -path $\omega = \omega_1 \omega_2 \dots \omega_n \in \Omega_n$ is called an *elementary cylinder set* and is denoted by $\text{cyl}(\omega)$. Thus,

$$\text{cyl}(\omega) = \{\omega' \in \Omega : \omega' = \omega_1 \omega_2 \dots \omega_n \omega'_{n+1} \omega'_{n+2} \dots\}$$

For an element A of the power set 2^{Ω_n} we define the *cylinder set*

$$\text{cyl}(A) = \cup \{\text{cyl}(\omega) : \omega \in A\}$$

Thus, $\text{cyl}(A)$ is the set of paths whose initial n -paths are elements of A . We use the notation

$$\mathcal{A}_n = \{\text{cyl}(A) : A \in 2^{\Omega_n}\}$$

It is easy to check that the cylinder sets form an increasing sequence $\mathcal{A}_1 \subseteq \mathcal{A}_2 \subseteq \dots$ of algebras on Ω and hence $\mathcal{C}(\Omega) = \cup \mathcal{A}_n$ is an algebra of subsets of

Ω . Letting \mathcal{A} be the σ -algebra generated by $\mathcal{C}(\Omega)$ we have that (Ω, \mathcal{A}) forms a measurable space.

If x produces y in r isomorphic ways, we say that the *multiplicity* of $x \rightarrow y$ is r and we write $m(x \rightarrow y) = r$. For example, in Figure 1, $m(x_3 \rightarrow x_6) = 2$. Let $c = (c_0, c_1, \dots)$ be a sequence of nonnegative numbers (called *coupling constants* [6, 9]). For $r, s \in \mathbb{N}$ with $r \leq s$, we define

$$\lambda_c(s, r) = \sum_{k=r}^s \binom{s-r}{k-r} c_k = \sum_{k=0}^{s-r} \binom{s-r}{k} c_{r+k}$$

For $x \in \mathcal{P}_n$, $y \in \mathcal{P}_{n+1}$ with $x \rightarrow y$ we define the *transition probability*

$$p_c(x \rightarrow y) = m(x \rightarrow y) \frac{\lambda_c(\alpha, \pi)}{\lambda_c(n, 0)}$$

where α is the number of ancestors and π the number of parents of the adjoined maximal element in y that produces y from x . It is shown in [6, 9] that $p_c(x \rightarrow y)$ is a probability distribution in that it satisfies the Markov-sum rule

$$\sum \{p_c(x \rightarrow y) : y \in x \rightarrow\} = 1$$

The distribution $p_c(x \rightarrow y)$ is essentially the most general that is consistent with principles of causality and covariance [6, 9]. It is hoped that other theoretical principles or experimental data will determine the coupling constants. One suggestion is to take $c_k = 1/k!$ [7]. The case $c_k = c^k$ for some $c > 0$ has been previously studied and is called a *percolation dynamics* [5, 6, 8].

The set \mathcal{P} together with the set of transition probabilities $p_c(x \rightarrow y)$ forms a *classical sequential growth process* (CSGP) which we denote by (\mathcal{P}, p_c) [5, 6, 7, 8]. It is clear that (\mathcal{P}, p_c) is a Markov chain and as usual we define the probability of an n -path $\omega = \omega_1 \omega_2 \cdots \omega_n$ by

$$p_c^n(\omega) = p(\omega_1 \rightarrow \omega_2) p(\omega_2 \rightarrow \omega_3) \cdots p(\omega_{n-1} \rightarrow \omega_n)$$

In this way $(\Omega_n, 2^{\Omega_n}, p_c^n)$ becomes a probability space where we define

$$p_c^n(A) = \sum \{p_c^n(\omega) : \omega \in A\}$$

for all $A \in 2^{\Omega_n}$. The probability of a causet $x \in \mathcal{P}_n$ is

$$p_c^n(x) = \sum \{p_c^n(\omega) : \omega \in \Omega_n, \omega_n = x\}$$

Of course, $x \mapsto p_c^n(x)$ is a probability measure on \mathcal{P}_n and we have

$$\sum_{x \in \mathcal{P}_n} p_c^n(x) = 1$$

For $A \in \mathcal{C}(\Omega)$ of the form $A = \text{cyl}(A_1)$, $A_1 \in 2^{\Omega_n}$, we define $p_c(A) = p_c^n(A)$. It is easy to check that p_c is a well-defined probability measure on the algebra $\mathcal{C}(\Omega)$. It follows from the Kolmogorov extension theorem that p_c has a unique extension to a probability measure ν_c on the σ -algebra \mathcal{A} . We conclude that $(\Omega, \mathcal{A}, \nu_c)$ is a probability space, the increasing sequence of subalgebras \mathcal{A}_n generate \mathcal{A} and that the restriction $\nu_c|_{\mathcal{A}_n} = p_c^n$.

We now “quantize” the CSGP (\mathcal{P}, ν_c) to obtain a quantum sequential growth process (QSGP). Let $H = L_2(\Omega, \mathcal{A}, \nu_c)$ be the *path Hilbert space* and $H_n = L_2(\Omega, \mathcal{A}_n, p_c^n)$ the *n-path Hilbert space*, $n = 1, 2, \dots$. Then $H_1 \subseteq H_2 \subseteq \dots$ forms an increasing sequence of closed subspaces of H . A bounded operator T on H_n will also be considered as a bounded operator on H by defining $Tf = 0$ for every $f \in H_n^\perp$. We denote the characteristic function χ_Ω of Ω by 1. Of course, $1 \in H$, $\|1\| = 1$ and $\langle 1, f \rangle = \int f d\nu_c$ for every $f \in H$. A *q-probability operator* is a bounded positive operator that satisfies $\langle \rho 1, 1 \rangle = 1$. Denote the set of *q-probability operators* on H and H_n by $\mathcal{Q}(H)$ and $\mathcal{Q}(H_n)$, respectively. A sequence $\rho_n \in \mathcal{Q}(H_n)$, $n = 1, 2, \dots$, is *consistent* if

$$\langle \rho_{n+1} \chi_B, \chi_A \rangle = \langle \rho_n \chi_B, \chi_A \rangle$$

for all $A, B \in \mathcal{A}_n$. A consistent sequence $\rho_n \in \mathcal{Q}(H_n)$, $n = 1, 2, \dots$, is called a *discrete quantum process* (DQP). A DQP $\rho_n \in \mathcal{Q}(H_n)$ on a CSGP (Ω, ν_c) is called a *quantum sequential growth process* (QSGP) [3, 4].

A rank 1 element of $\mathcal{Q}(H)$ is called a *pure q-probability operator*. Thus $\rho \in \mathcal{Q}(H)$ is pure if and only if ρ has the form $\rho = |\psi\rangle\langle\psi|$ for some $\psi \in H$ satisfying $|\langle 1, \psi \rangle| = 1$ or equivalently $|\int \psi d\nu_c| = 1$. We then call ψ a *q-probability vector* and we denote the set of pure *q-probability operators* by $\mathcal{Q}_p(H)$. A QSGP ρ_n is a *pure QSGP* if $\rho_n \in \mathcal{Q}_p(H_n)$, $n = 1, 2, \dots$.

For $\rho_n \in \mathcal{Q}(H_n)$ we define the *n-decoherence functional* $D_n: \mathcal{A} \times \mathcal{A} \rightarrow \mathbb{C}$ by

$$D_n(A, B) = \langle \rho_n \chi_B, \chi_A \rangle$$

and this is a measure of the interference between events A and B when the process is described by ρ_n [3, 4]. We define the map $\mu_n: \mathcal{A} \rightarrow \mathbb{R}^+$ by

$$\mu_n(A) = D_n(A, A) = \langle \rho_n \chi_A, \chi_A \rangle$$

where $\mu_n(A)$ gives the quantum propensity for the occurrence of the event A when the process is described by ρ_n . Although μ_n does not give a probability because it is not additive, it does satisfy the *grade-2 additivity condition*: if $A, B, C \in \mathcal{A}$ are mutually disjoint, then

$$\begin{aligned} \mu_n(A \cup B \cup C) \\ = \mu_n(A \cup B) + \mu_n(A \cup C) + \mu_n(B \cup C) - \mu_n(A) - \mu_n(B) - \mu_n(C) \end{aligned} \quad (2.1)$$

Notice that $\mu_n(\Omega) = 1$ and if ρ_n is a DQP then $\mu_{n+1}(A) = \mu_n(A)$ for all $A \in \mathcal{A}_n$. Since $\mu_n(A) = \|\rho_n^{1/2} \chi_A\|^2$, we conclude that μ_n is the squared norm of a vector-valued measure $A \mapsto \rho_n^{1/2} \chi_A$. In particular, if $\rho_n = |\psi_n\rangle\langle\psi_n|$ is a pure QSGP, then $\mu_n(A) = |\langle\psi_n, \chi_A\rangle|^2$ so μ_n is the squared modulus of the complex-valued measure $A \mapsto \langle\psi_n, \chi_A\rangle$.

If ρ_n is a QSGP, we say that $A \in \mathcal{A}$ is *suitable* if $\lim \mu_n(A)$ exists and is finite and in this case we define $\mu(A)$ to be the limit. We denote the collection of suitable sets by $\mathcal{S}(\rho_n)$. If $A \in \mathcal{A}_n$, then $\lim \mu_n(A) = \mu_n(A)$ so $A \in \mathcal{S}(\rho_n)$ and $\mu(A) = \mu_n(A)$. In particular, $\Omega \in \mathcal{S}(\rho_n)$ and $\mu(\Omega) = 1$. This shows that the algebra $\mathcal{C}(\Omega) \subseteq \mathcal{S}(\rho_n)$. In general, $\mathcal{S}(\rho_n) \neq \mathcal{A}$ and μ does not have a well-behaved extension from $\mathcal{C}(\Omega)$ to all of \mathcal{A} [2, 3, 5]. A subset \mathcal{B} of \mathcal{A} is a *quadratic algebra* if $\phi, \Omega \in \mathcal{B}$ and whenever $A, B, C \in \mathcal{B}$ are mutually disjoint with $A \cup B, A \cup C, B \cup C \in \mathcal{B}$, we have $A \cup B \cup C \in \mathcal{B}$. For a quadratic algebra \mathcal{B} , a *q-measure* is a map $\mu_0: \mathcal{B} \rightarrow \mathbb{R}^+$ that satisfies the grade-2 additivity condition (2.1). Of course, an algebra of sets is a quadratic algebra and we conclude that $\mu_n: \mathcal{A} \rightarrow \mathbb{R}^+$ is a *q-measure*. It is not hard to show that $\mathcal{S}(\rho_n)$ is a quadratic algebra and $\mu: \mathcal{S}(\rho_n) \rightarrow \mathbb{R}^+$ is a *q-measure* that extends $\mu_n: \mathcal{C}(\Omega) \rightarrow \mathbb{R}^+$.

For a QSGP ρ_n , we call ρ_n the *local operators* and μ_n the *local q-measures* for the process. If $\rho = \lim \rho_n$ exists in the strong operator topology, then $\rho \in \mathcal{Q}(H)$ and we call ρ the *global operator* for the process. If the global operator ρ exists, then $\hat{\mu}(A) = \langle \rho \chi_A, \chi_A \rangle$ is a (continuous) *q-measure* on \mathcal{A} that extends μ_n , $n = 1, 2, \dots$. Unfortunately, the global operator does not exist in general, so we must work with the local operators [2, 3]. In this case, we still have the *q-measure* μ on the quadratic algebra $\mathcal{S}(\rho_n) \subseteq \mathcal{A}$ that extends μ_n , $n = 1, 2, \dots$.

As with the coupling constants c_n for a CSGP we will need additional theoretical principles or experimental data to determine the local operators $\rho_n \in \mathcal{Q}(H_n)$ for a QSGP. However, we can still make some observations even with our limited knowledge. For example, suppose we are interested in the

quantum propensity $\mu(A)$ of the event $A \in \mathcal{S}(\rho_n)$. Assume it is known that the classical probability $\nu_c(A)$ of A occurring is small or even zero. Then the vector $\chi_A \in H$ has small norm $\|\chi_A\| = \nu_c(A)^{1/2}$. We then have

$$\mu_n(A) = \langle \rho_n \chi_A, \chi_A \rangle \leq \|\rho_n\| \|\chi_A\|^2 = \|\rho_n\| \nu_c(A)$$

If $\nu_c(A) = 0$ then $\mu_n(A) = 0$, $n = 1, 2, \dots$, so $\mu(A) = 0$. If $\|\rho_n\|$ are uniformly bounded $\|\rho_n\| \leq M$, $n = 1, 2, \dots$, (which frequently happens) then $\mu(A) \leq M \nu_c(A)$ which is small if M is reasonable. Finally, if A happens to be in \mathcal{A}_n for some n , then

$$\mu(A) = \mu_n(A) \leq \|\rho_n\| \nu_c(A)$$

which again is small if $\|\rho_n\|$ is reasonable.

This section closes with a simple method for constructing a QSGP from a CSGP (\mathcal{P}, p_c) . Although there are more general methods [4], the present one is instructive because it generates a quantum Markov chain. For all $x \in \mathcal{P}_n$, $y \in \mathcal{P}_{n+1}$ with $x \rightarrow y$, let $\alpha(x \rightarrow y) \in \mathbb{C}$ satisfy

$$\sum_{y \in x \rightarrow} \alpha(x \rightarrow y) p_c(x \rightarrow y) = 1$$

We call $\alpha(x \rightarrow y)$ a *transition amplitude from x to y* and for $\omega = \omega_1 \omega_2 \cdots \omega_n \in \Omega_n$ we define the *amplitude*

$$\alpha(\omega) = \alpha(\omega_1 \rightarrow \omega_2) \alpha(\omega_2 \rightarrow \omega_3) \cdots \alpha(\omega_{n-1} \rightarrow \omega_n)$$

We next introduce the vector $\psi_n \in H$ given by

$$\psi_n = \sum_{\omega \in \Omega_n} \alpha(\omega) \chi_{\text{cyl}(\omega)}$$

It is important to notice that $\psi_n \in H_n$, $n = 1, 2, \dots$.

Theorem 2.1. *The sequence $\rho_n = |\psi_n\rangle\langle\psi_n|$ gives a pure QSGP.*

Proof. To show that ψ_n is a q -probability vector we have

$$\begin{aligned}
\langle 1, \psi_n \rangle &= \left\langle \sum_{\omega' \in \Omega_n} \chi_{\text{cyl}(\omega')}, \sum_{\omega \in \Omega_n} \alpha(\omega) \chi_{\text{cyl}(\omega)} \right\rangle = \sum_{\omega \in \Omega_n} \alpha(\omega) p_c^n(\omega) \\
&= \sum_{\omega \in \Omega_n} \alpha(\omega_1 \rightarrow \omega_2) p_c(\omega_1 \rightarrow \omega_2) \cdots \alpha(\omega_{n-1} \rightarrow \omega_n) p_c^n(\omega_{n-1} \rightarrow \omega_n) \\
&= \sum_{\omega \in \Omega_{n-1}} \alpha(\omega_1 \rightarrow \omega_2) p_c(\omega_1 \rightarrow \omega_2) \cdots \alpha(\omega_{n-2} \rightarrow \omega_{n-1}) p_c^n(\omega_{n-2} \rightarrow \omega_{n-1}) \\
&\quad \vdots \\
&= \sum_{\omega \in \Omega_2} \alpha(\omega_1 \rightarrow \omega_2) p_c^n(\omega_1 \rightarrow \omega_2) = 1
\end{aligned}$$

To show that ρ_n is a consistent sequence we use the notation ωx for $\omega_1 \omega_2 \cdots \omega_n x \in \Omega_{n+1}$ where $\omega = \omega_1 \omega_2 \cdots \omega_n \in \Omega_n$ and $x \in \mathcal{P}_{n+1}$ with $\omega_n \rightarrow x$ to obtain

$$\begin{aligned}
\sum_{x \in \omega_n \rightarrow} \langle \chi_{\text{cyl}(\omega x)}, \psi_{n+1} \rangle &= \sum_{x \in \omega_n \rightarrow} \left\langle \chi_{\text{cyl}(\omega x)}, \sum_{\omega' \in \Omega_{n+1}} \alpha(\omega') \chi_{\text{cyl}(\omega')} \right\rangle \\
&= \sum_{x \in \omega_n \rightarrow} \alpha(\omega x) p_c^n(\omega x) \\
&= \sum_{x \in \omega_n \rightarrow} \alpha(\omega) \alpha(\omega_n \rightarrow x) p_c^n(\omega) p_c(\omega_n \rightarrow x) \\
&= \alpha(\omega) p_c^n(\omega) = \langle \chi_{\text{cyl}(\omega)}, \psi_n \rangle
\end{aligned}$$

For $\omega, \omega' \in \Omega_n$, it follows that

$$\begin{aligned}
D_{n+1}(\text{cyl}(\omega), \text{cyl}(\omega')) &= \langle |\psi_{n+1}\rangle \langle \psi_{n+1}| \chi_{\text{cyl}(\omega')}, \chi_{\text{cyl}(\omega)} \rangle \\
&= \langle \chi_{\text{cyl}(\omega')}, \psi_{n+1} \rangle \langle \psi_{n+1}, \chi_{\text{cyl}(\omega)} \rangle \\
&= \sum_{x' \in \omega'_n \rightarrow} \langle \chi_{\text{cyl}(\omega' x')}, \psi_{n+1} \rangle \sum_{x \in \omega_n \rightarrow} \langle \psi_{n+1}, \chi_{\text{cyl}(\omega x)} \rangle \\
&= \langle \chi_{\text{cyl}(\omega')}, \psi_n \rangle \langle \psi_n, \chi_{\text{cyl}(\omega)} \rangle \\
&= D_n(\text{cyl}(\omega), \text{cyl}(\omega'))
\end{aligned}$$

For $A, B \in \mathcal{A}_n$ we have

$$D_n(A, B) = \sum \{ D_n(\text{cyl}(\omega), \text{cyl}(\omega')) : \omega, \omega' \in \mathcal{A}_n, \text{cyl}(\omega) \subseteq A, \text{cyl}(\omega') \subseteq B \}$$

and the result follows. \square

The decoherence functional $D_n: \mathcal{A} \times \mathcal{A} \rightarrow \mathbb{C}$ and the q -measure $\mu_n: \mathcal{A} \rightarrow \mathbb{R}^+$ corresponding to $\rho_n = |\psi_n\rangle\langle\psi_n|$ are given by

$$\begin{aligned} D_n(A, B) &= \langle\psi_n, \chi_A\rangle\langle\chi_B, \psi_n\rangle \\ \mu_n(A) &= |\langle\chi_A, \psi_n\rangle|^2 \end{aligned}$$

Notice that μ_n is the modulus squared of the complex measure $\lambda_n(A) = \langle\chi_A, \psi_n\rangle$. Moreover, we have for all $A \in \mathcal{A}$ that

$$\begin{aligned} \lambda_n(A) &= \left\langle \chi_A, \sum_{\omega \in \Omega_n} \alpha(\omega) \chi_{\text{cyl}(\omega)} \right\rangle = \sum_{\omega \in \Omega_n} \alpha(\omega) \langle\chi_A, \chi_{\text{cyl}(\omega)}\rangle \\ &= \sum_{\omega \in \Omega_n} \alpha(\omega) \nu_c[A \cap \text{cyl}(\omega)] \end{aligned}$$

In particular, if $A \in \mathcal{A}$ then $A = \text{cyl}(A_1)$ for some $A_1 \in 2^{\Omega_n}$ and we have

$$\lambda_n(A) = \sum_{\omega \in A_1} \alpha(\omega) p_c^n(\omega)$$

Moreover, for $\omega = \omega_1 \omega_2 \cdots \in \Omega$ we obtain

$$\mu_n(\{\omega\}) = |\alpha(\omega_1 \omega_2 \cdots \omega_n) p_c^n(\omega_1 \omega_2 \cdots \omega_n)|^2$$

Example 1. The simplest transition amplitude is $\alpha(x \rightarrow y) = 1$ for every x and y with $x \rightarrow y$. Then $\alpha(\omega) = 1$ for every $\omega \in \Omega_n$ and

$$\psi_n = \sum_{\omega \in \Omega_n} \chi_{\text{cyl}(\omega)} = 1$$

$n = 1, 2, \dots$. The corresponding decoherence functional becomes

$$D_n(A, B) = \langle 1, \chi_A \rangle \langle \chi_B, 1 \rangle = \nu_c(A) \nu_c(B)$$

and $\mu_n(A) = \nu_c(A)^2$. Thus, $\mathcal{S}(\rho_n) = \mathcal{A}$ and $\mu(A) = \nu_c(A)^2$ is the classical probability squared. Moreover, $|1\rangle\langle 1|$ is the global operator.

Example 2. Assume that $p(x \rightarrow y) \neq 0$ for every x and y with $x \rightarrow y$ and define the transition amplitude

$$\alpha(x \rightarrow y) = \frac{1}{p(x \rightarrow y) |x \rightarrow|}$$

where $|x \rightarrow|$ is the cardinality of $x \rightarrow$. For $\omega = \omega_1 \omega_2 \cdots \omega_n \in \Omega_n$, letting

$$\beta_n(\omega) = \frac{1}{|\omega_1 \rightarrow| |\omega_2 \rightarrow| \cdots |\omega_{n-1} \rightarrow|}$$

we have $\alpha(\omega) = 1/p_c^n(\omega)\beta_n(\omega)$. If $A \in 2^{\Omega_n}$, then

$$\mu(\text{cyl}(A)) = \mu_n(\text{cyl}(A)) = \left| \sum_{\omega \in A} \beta_n(\omega) \right|^2$$

If $\omega = \omega_1 \omega_2 \cdots \in \Omega$ then $\mu_n(\{\omega\}) = \beta_n(\omega)^2$. Since $\lim \beta_n(\omega) = 0$, we conclude that $\{\omega\} \in \mathcal{S}(\rho_n)$ and $\mu(\{\omega\}) = 0$. It follows that if $A \subseteq \Omega$ with $|A| < \infty$, then $A \in \mathcal{S}(\rho_n)$ and $\mu(A) = 0$. We conjecture that $\mathcal{S}(\rho_n) \neq \mathcal{A}$ and there is no global operator.

3 Matter-Antimatter

Figure 1 illustrates the first four steps of a CSGP. The first four levels are complete but for lack of space, level five is not. The numbers on the arrows designate the multiplicity of the transition. Except for x_1 we classify the causets in terms of three types. The vertical rectangles on the left correspond to antimatter causets, the circles in the middle correspond to mixed coasts and the horizontal rectangles on the right correspond to matter causets. We whimsically call x_1 “neutrino,” x_2 “positron,” x_3 “electron” and x_4 – x_8 “quarks” (there are six quarks, counting multiplicity). In this way of thinking about matter-antimatter, we view a causet not only as a scaffolding for the geometry of a universe but also a placement of masses. Although there is some symmetry between antimatter and matter causets, if we take multiplicity into account there is a definite asymmetry. Since there is much more multiplicity on the matter side, this already indicates that this side might be more probable, but we shall discuss this later. We first present a rigorous definition of our classification scheme.

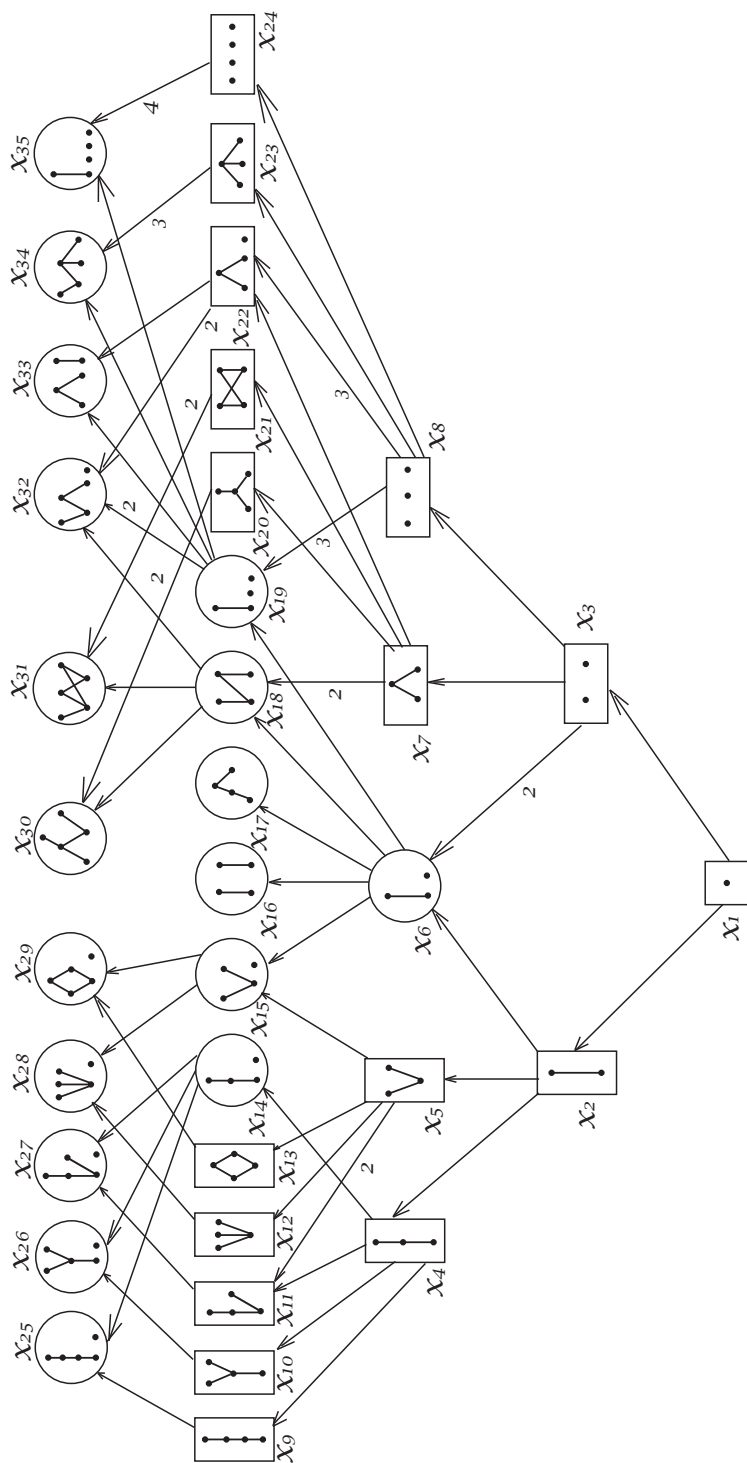


Figure 1

In our classification scheme we consider x_2 as the source of antimatter and x_3 as the source of matter. We call a causet y a *product* of a causet x if $x \neq x_1$ and either $y = x$ or there is a path containing x and subsequently y . If x is a product of x_2 and not x_3 , then x is an *antimatter causet*. If x is a product of x_3 and not x_2 , then x is a *matter causet*. If x is a product of x_6 , then x is a *mixed causet*. Notice that x_1 is not classified. We denote the set of antimatter, matter and mixed causets by ANT, MAT and MIX, respectively. We also use the notation $\text{ANT}_n = \text{ANT} \cap \mathcal{P}_n$, $\text{MAT}_n = \text{MAT} \cap \mathcal{P}_n$, $\text{MIX}_n = \text{MIX} \cap \mathcal{P}_n$, $n = 1, 2, \dots$. As examples we have $x_2 \in \text{ANT}_2$, $x_3 \in \text{MAT}_2$, $x_4, x_5 \in \text{ANT}_3$, $x_6 \in \text{MIX}_3$, $x_7, x_8 \in \text{MAT}_3$.

We call an element of a causet a *vertex*. A vertex $a \in x$ is *minimal* if there is no $b \in x$ with $b < a$. A vertex is a *single-parent child* if it is the child of precisely one parent and is a *multiple-parent child* if it is the child of more than one parent. No matter how complicated it is, the next result gives a method of quickly determining whether a causet is in ANT, MAT or MIX.

Theorem 3.1. (i) A causet x is a product of x_2 if and only if x has a minimal vertex with a single-parent child. (ii) A causet x is a product of x_3 if and only if x has at least two minimal vertices. (iii) $x \in \text{MIX}$ if and only if x has two (or more) minimal vertices at least one of which has a single-parent child. (iv) $x \in \text{ANT}$ if and only if x has only one minimal vertex and $|x| \geq 2$. (v) $x \in \text{MAT}$ if and only if x has at least two minimal vertices and no minimal vertex has a single-parent child

Proof. (i) x_2 has a minimal vertex with a single-parent child and adjoining a maximal vertex to x_2 does not change this fact. If x is a product of x_2 it follows by induction that x has a minimal vertex with a single-parent child. Conversely, suppose x has a minimal vertex a with a single-parent child b . If $x = x_2$ then x is a product of x_2 and we are finished. Otherwise, x has a maximal vertex $c \neq a, b$. We remove c and we still have a and b in the remaining causet. We continue this process until we obtain $x_2 = \{a, b\}$. Reversing this process gives a path containing x_2 and subsequently x . Hence, x is a product of x_2 .

(ii) x_3 has two minimal vertices and adjoining a maximal vertex to x_3 does not change this fact. As in (i), we conclude that if x is a product of x_3 , then x has at least two minimal vertices. Conversely, suppose x has two minimal vertices $c \neq a, b$. As before, we remove c and continue this process until we obtain $x_3 = \{a, b\}$. As in (i) we conclude that x is a product of x_3 .

(iii) If $x \in \text{MIX}$ then x is a product of x_6 and hence x is a product of both x_2

and x_3 . Applying (i) and (ii) we conclude that x has two minimal vertices at least one of which has a single-parent child. Conversely, suppose x has two minimal vertices a, b and a has a single-parent child c . If c and b are the only maximal vertices of x , then $x = x_6$ and we are finished. Otherwise, x has a maximal vertex other than c or b and we remove it. The resulting causet y still has the minimal vertices a, b and a still has the single-parent child. Continue this process until we arrive at x_6 . Reversing the process shows that x is a product of x_6 .

(iv) If $x \in \text{ANT}$ then $|x| \geq 2$ and $x \notin \text{MAT}$ so by (ii) x has only one minimal vertex. Conversely, suppose x has only one minimal vertex a and $|x| \geq 2$. By (ii) $x \notin \text{MAT}$. Now a must have a child b because otherwise, since $|x| \geq 2$ there would be another minimal vertex. If b had a parent c with $c \neq a$, then c is either minimal or there is a minimal element d having c as a product. The first case contradicts the fact that a is the only minimal vertex. In the second case, $d \neq a$ because b is a child of a and not of d . This again contradicts the fact that a is the only minimal vertex. Hence, b is a single-parent child and by (i), $x \in \text{ANT}$.

(v) If $x \in \text{MAT}$, then $x \notin \text{ANT}$ so by (iv) x must have at least two minimal vertices. Moreover, by (i) no minimal vertex has a single-parent child. Conversely, suppose x has two minimal vertices and no minimal vertex has a single-parent child. By (iv), $x \notin \text{ANT}$. By (ii), x is a product of x_3 so $x \in \text{MAT}$. \square

Corollary 3.2. *A causet $x \in \text{MIX}$ is and only if x is a product of x_2 and x_3 .*

Proof. If $x \in \text{MIX}$ then clearly, x is a product of x_2 and x_3 . Conversely, suppose x is a product of x_2 and x_3 . By Theorem 3.1 (i), (ii) and (iii), $x \in \text{MIX}$. \square

Corollary 3.3. (i) ANT_n , MAT_n and MIX_n are mutually disjoint and

$$\mathcal{P}_n = \text{ANT}_n \cup \text{MIX}_n \cup \text{MAT}_n$$

(ii) ANT , MAT and MIX are mutually disjoint and

$$\mathcal{P} = \text{ANT} \cup \text{MIX} \cup \text{MAT}$$

Proof. This follows from Corollary 3.2. \square

The next corollary shows that if $x \in \text{ANT} \cup \text{MAT}$ and $y \in \text{MIX}$ with $x \rightarrow y$, then $p(x \rightarrow y)$ has a simple form

Corollary 3.4. (i) *If $x \in \text{ANT}$, $y \in \text{MIX}$ with $x \rightarrow y$, then*

$$p(x \rightarrow y) = \frac{m(x \rightarrow y)c_0}{\lambda_c(n, 0)}$$

(ii) *If $x \in \text{MAT}$, $y \in \text{MIX}$ with $x \rightarrow y$, then*

$$p(x \rightarrow y) = \frac{m(x \rightarrow y)c_1}{\lambda_c(n, 0)}$$

Proof. Suppose x produces y by adjoining the maximal vertex a to x .

(i) By Theorem 3.1 (iv), x has only one minimal vertex b and, of course, b is still minimal in y . Since $y \in \text{MIX}$, by Theorem 3.1 (iii) y must have at least two minimal elements so a is maximal and minimal in y . Thus, a is isolated in y so a has no parents and no ancestors. Hence, $\lambda_c(\alpha, \pi) = c_0$ and the result follows.

(ii) By Theorem 3.1 (v) x has minimal vertices b_1, \dots, b_n , $n \geq 2$, none of which has a single-parent child. Since $y \in \text{MIX}$ by Theorem 3.1 (iii), a must be a single-parent child of precisely one of the b_1, \dots, b_n in y . Thus, x has one parent and one ancestor so $\lambda_c(\alpha, \pi) = c_1$ and the result follows. \square

There are indications that in the partition of \mathcal{P}_n in Corollary 3.3 (i) that MIX_n dominates ANT_n and MAT_n both numerically and probabilistically for large n . One reason for this is that if $x \rightarrow y$ then

- (1) $y \in \text{MIX}$ when $x \in \text{MIX}$
- (2) $y \in \text{ANT} \cup \text{MIX}$ when $x \in \text{ANT}$
- (3) $y \in \text{MAT} \cup \text{MIX}$ when $x \in \text{MAT}$.

The next result shows that the part of MIX in (2) and (3) is nonempty. This shows that MIX continues to grow compared to ANT and MAT as n increases.

Theorem 3.5. (i) *If $x \in \text{ANT}$, then x produces precisely one offspring in MIX .* (ii) *If $x \in \text{MAT}$ has m minimal vertices, then x produces precisely m offspring (including multiplicity) in MIX .*

Proof. (i) By Theorem 3.1 (iv), x has precisely one minimal vertex and this vertex must have a single-parent child. If we adjoin an isolated vertex to x then by Theorem 3.1 (iii) the produced causet is mixed. Also, this is the only offspring of x . (ii) By Theorem 3.1 (v), none of the minimal vertices has a single-parent child. By adjoining a single-parent child to one of the minimal vertices, x produces a mixed offspring according to Theorem 3.1 (iii). This can be done in precisely m ways. \square

A path $\omega = \omega_1\omega_2\cdots \in \Omega$ is an *antimatter* (*matter*) *path* if $\omega_i \in \text{ANT}(\text{MAT})$, $i = 2, 3, \dots$. A path $\omega = \omega_1\omega_2\cdots \in \Omega$ is *mixed* if $\omega_i \in \text{MIX}$ for some $i \in \{3, 4, \dots\}$. Of course, if $\omega_i \in \text{MIX}$ then $\omega_{i+1}, \omega_{i+2}, \dots \in \text{MIX}$. We denote the sets of antimatter, matter and mixed paths by Ω^{ANT} , Ω^{MAT} , Ω^{MIX} , respectively. By restricting i to $1, 2, \dots, n$ we obtain the sets of n -paths $\Omega_n^{\text{ANT}}, \Omega_n^{\text{MAT}}, \Omega_n^{\text{MIX}} \in \Omega_n$ in the natural way. It is clear that Ω^{ANT} , Ω^{MAT} and Ω^{MIX} are mutually disjoint and

$$\Omega = \Omega^{\text{ANT}} \cup \Omega^{\text{MIX}} \cup \Omega^{\text{MAT}}$$

Similarly, Ω_n^{ANT} , Ω_n^{MAT} and Ω_n^{MIX} are mutually disjoint and

$$\Omega_n = \Omega_n^{\text{ANT}} \cup \Omega_n^{\text{MIX}} \cup \Omega_n^{\text{MAT}}$$

Since $\text{cyl}(\Omega_{n+1}^{\text{ANT}}) \subseteq \text{cyl}(\Omega_n^{\text{ANT}})$, $n = 2, 3, \dots$, and $\Omega^{\text{ANT}} = \bigcap \text{cyl}(\Omega_n^{\text{ANT}})$ we conclude that $\Omega^{\text{ANT}} \in \mathcal{A}$ and

$$\nu_c(\Omega^{\text{ANT}}) = \lim_{n \rightarrow \infty} p_c^n(\Omega_n^{\text{ANT}})$$

Similarly, $\text{cyl}(\Omega_{n+1}^{\text{MAT}}) \subseteq \text{cyl}(\Omega_n^{\text{MAT}})$, $n = 2, 3, \dots$, and $\Omega^{\text{MAT}} = \bigcap \text{cyl}(\Omega_n^{\text{MAT}})$ so that $\Omega^{\text{MAT}} \in \mathcal{A}$ and

$$\nu_c(\Omega^{\text{MAT}}) = \lim_{n \rightarrow \infty} p_c^n(\Omega_n^{\text{MAT}})$$

Moreover, $\text{cyl}(\Omega_{n+1}^{\text{MIX}}) \subseteq \text{cyl}(\Omega_n^{\text{MIX}})$, $n = 2, 3, \dots$, and $\Omega^{\text{MIX}} = \bigcup \text{cyl}(\Omega_n^{\text{MIX}})$ so that $\Omega^{\text{MIX}} \in \mathcal{A}$ and

$$\nu_c(\Omega^{\text{MIX}}) = \lim_{n \rightarrow \infty} p_c^n(\Omega_n^{\text{MIX}})$$

4 Probabilities and Propensities

This section computes some probabilities and notes some q -propensities and trends that seem to hold. One of the few probabilities that we have been

able to compute precisely is $p_c^n(\Omega_n^{\text{ANT}})$. Since $\Omega_2^{\text{ANT}} = \{x_1x_2\}$ we have

$$p_c^2(\Omega_2^{\text{ANT}}) = p_c(x_1 \rightarrow x_2) = \frac{c_1}{c_0 + c_1}$$

By Corollary 3.4 (i) we have

$$\begin{aligned} p_c^3(\Omega_3^{\text{ANT}}) &= p_c^3(x_1x_2x_4) + p_c^3(x_1x_2x_5) \\ &= p_c(x_1 \rightarrow x_2) [p_c(x_2 \rightarrow x_4) + p_c(x_2 \rightarrow x_5)] \\ &= p_c(x_1 \rightarrow x_2) [1 - p_c(x_2 \rightarrow x_6)] = \frac{c_1}{c_0 + c_1} \left[1 - \frac{c_0}{c_0 + 2c_1 + c_2} \right] \end{aligned}$$

Since

$$\Omega_4^{\text{ANT}} = \{x_1x_2x_4x_9, x_1x_2x_4x_{10}, x_1x_2x_4x_{11}, x_1x_2x_5x_{11}, x_1x_2x_5x_{12}, x_1x_2x_5x_{13}\}$$

by Corollary 3.4(i) we have

$$\begin{aligned} p_c^4(\Omega_4^{\text{ANT}}) &= p_c(x_1 \rightarrow x_2) \{p_c(x_2 \rightarrow x_4) [p_c(x_4 \rightarrow x_9) + p_c(x_4 \rightarrow x_{10}) + p_c(x_4 \rightarrow x_{11})] \\ &\quad + p_c(x_2 \rightarrow x_5) [p_c(x_5 \rightarrow x_{11}) + p_c(x_5 \rightarrow x_{12}) + p_c(x_5 \rightarrow x_{13})]\} \\ &= p_c(x_1 \rightarrow x_2) \{p_c(x_2 \rightarrow x_4) [1 - p_c(x_4 \rightarrow x_{14})] \\ &\quad + p_c(x_2 \rightarrow x_5) [1 - p_c(x_5 \rightarrow x_{15})]\} \\ &= p_c(x_1 \rightarrow x_2) [1 - p_c(x_2 \rightarrow x_6)] [1 - p_c(x_4 \rightarrow x_{14})] \\ &= \frac{c_1}{c_0 + c_1} \left(1 - \frac{c_0}{c_0 + 2c_1 + c_2} \right) \left(1 - \frac{c_0}{c_0 + 3c_1 + 3c_2 + c_3} \right) \end{aligned}$$

Continuing by induction we obtain

$$p_c^n(\Omega_n^{\text{ANT}}) = \frac{c_0}{c_0 + c_1} \left(1 - \frac{c_0}{c_0 + 2c_1 + c_2} \right) \cdots \left(1 - \frac{c_0}{\sum_{i=0}^{n-1} \binom{n-1}{i} c_i} \right) \quad (4.1)$$

The sequence (4.1) converges to $\nu_c(\Omega^{\text{ANT}})$.

To get an idea of the probabilities $\nu_c(\Omega^{\text{ANT}})$ consider a percolation dynamics with $c_k = c^k$, $c > 0$. Then (4.1) becomes

$$p_c^n(\Omega_n^{\text{ANT}}) = \frac{c_1}{1+c} \left[1 - \frac{1}{(1+c)^2} \right] \left[1 - \frac{1}{(1+c)^3} \right] \cdots \left[1 - \frac{1}{(1+c)^{n-1}} \right] \quad (4.2)$$

Table 1 gives $\nu_c(\Omega^{\text{ANT}})$ for various values of c . Notice that, as a function of c , $\nu_c(\Omega^{\text{ANT}})$ increases monotonically until it reaches a maximum of 0.3035 at about $c = 1.3$ and then decreases monotonically.

c	0.2	0.3	0.5	0.7	0.9	1.0	1.1	1.2
$\nu_c(\Omega^{\text{ANT}})$	0.00357	0.03122	0.1385	0.2264	0.2753	0.2888	0.2972	0.3018

c	1.3	1.4	1.5	1.6	2.0	3.0	4.0
$\nu_c(\Omega^{\text{ANT}})$	0.3035	0.3031	0.3012	0.2982	0.2801	0.2295	0.1901

Table 1

We suspect that for this model, the q -propensities will be smaller. For instance in Example 1, the maximum values of the q -propensity is

$$\mu(\Omega^{\text{ANT}}) = (0.3035)^2 = 0.09211$$

We next consider the factorial dynamics $c_k = 1/k!$. In this case (4.1) becomes

$$p_c^n(\Omega_n^{\text{ANT}}) = \frac{1}{2} \prod_{j=2}^{n-1} \left[1 - \frac{1}{\sum_{i=0}^j \binom{j}{i} \frac{1}{i!}} \right]$$

Table 2 gives $p_c^n(\Omega_n^{\text{ANT}})$ for various values of n .

n	10	20	50	100
$p_c^n(\Omega_n^{\text{ANT}})$	0.20793	0.19522	0.19363	0.19362

Table 2

According to Mathematica

$$\nu_c(\Omega^{\text{ANT}}) = \lim_{n \rightarrow \infty} p_c^n(\Omega_n^{\text{ANT}}) = 0.19361$$

We have found $\nu_c(\Omega^{\text{ANT}})$ for other coupling constants. For $c_k = 1/(k!)^k$, $c_k = 1/((k+1)^k)$, $c_k = 1/(k!)^{50}$ we have that $\nu_c(\Omega^{\text{ANT}})$ equals 0.093313, 0.032498, 0.0049751, respectively.

Due to multiplicities, the situation for $p_c^n(\Omega_n^{\text{MAT}})$ becomes much more complicated and we have not found this quantity precisely. However, we can point out some tendencies. Proceeding as we did for Ω_n^{ANT} we obtain

$$\begin{aligned} p_c^2(\Omega_2^{\text{MAT}}) &= \frac{c_0}{c_0 + c_1} \\ p_c^3(\Omega_3^{\text{MAT}}) &= \frac{c_0}{c_0 + c_1} \left[1 - \frac{2c_1}{c_0 + 2c_1 + c_2} \right] \\ p_c^4(\Omega_4^{\text{MAT}}) &= \frac{c_0}{c_0 + c_1} \left[1 - \frac{c_1}{c_0 + 2c_1 + c_2} \left(1 + \frac{2c_2 + 3c_0}{c_0 + 3c_1 + 3c_2 + c_3} \right) \right] \end{aligned}$$

and the terms become increasingly more complicated.

We now compare the first few terms of $p_c^n(\Omega_n^{\text{MAT}})$ with $p_c^n(\Omega_n^{\text{ANT}})$. For the percolation dynamics with $c = 1$ we have

$$\begin{aligned} p_c^2(\Omega_2^{\text{MAT}}) &= p_c^2(\Omega_2^{\text{ANT}}) = \frac{1}{2} \\ p_c^3(\Omega_3^{\text{MAT}}) &= \frac{1}{4}, \quad p_c^3(\Omega_3^{\text{ANT}}) = \frac{5}{14} \\ p_c^4(\Omega_4^{\text{MAT}}) &= \frac{19}{64}, \quad p_c^4(\Omega_4^{\text{ANT}}) = \frac{21}{64} \end{aligned}$$

For the factorial dynamics $c_k = 1/k!$ we have

$$\begin{aligned} p_c^2(\Omega_2^{\text{MAT}}) &= p_c^2(\Omega_2^{\text{ANT}}) = \frac{1}{2} \\ p_c^3(\Omega_3^{\text{MAT}}) &= \frac{3}{14}, \quad p_c^3(\Omega_3^{\text{ANT}}) = \frac{3}{8} \\ p_c^4(\Omega_4^{\text{MAT}}) &= \frac{61}{238}, \quad p_c^4(\Omega_4^{\text{ANT}}) = \frac{70}{238} \end{aligned}$$

The previous data indicates that the tendency is for $p_c^n(\Omega_n^{\text{MAT}}) < p_c^n(\Omega_n^{\text{ANT}})$. This is also consistent with Theorem 3.5 which shows that $|\Omega_n^{\text{MAT}}|$ is much smaller than $|\Omega_n^{\text{ANT}}|$ for large n . When this tendency holds, both $\nu_c(\Omega^{\text{MAT}})$ and $\nu_c(\Omega^{\text{ANT}})$ are small and this indicates that $\mu(\Omega^{\text{MAT}})$ and $\mu(\Omega^{\text{ANT}})$ are also likely to be small.

Example 3. This example gives further evidence for the tendency $p_c^n(\Omega_n^{\text{MAT}}) < p_c^n(\Omega_n^{\text{ANT}})$. Let $\omega_a^n \in \Omega_n$ be the extreme antimatter path $\omega_a^n = x_1 x_2 x_4 x_9 \cdots y$. Then

$$p_c^n(\omega_a^n) = \left(\frac{c_1}{c_0 + c_1} \right) \left(\frac{c_1 + c_2}{c_0 + 2c_1 + c_2} \right) \cdots \left(\frac{\sum_{j=1}^n \binom{n-1}{j-1} c_j}{\sum_{j=0}^n \binom{n}{j} c_j} \right)$$

If $\omega_m^n \in \Omega_n$ is the extreme matter path $\omega_m^n = x_1 x_3 x_8 x_{24} \cdots z$ we have

$$p_c^n(\omega_m^n) = \left(\frac{c_0}{c_0 + c_1} \right) \left(\frac{c_0}{c_0 + 2c_1 + c_2} \right) \cdots \left(\frac{c_0}{\sum_{j=0}^n \binom{n}{j} c_j} \right)$$

For the percolation dynamics $c_k = c^k$ we have

$$p_c^n(\omega_a^n) = \left(\frac{c}{1+c} \right)^n$$

$$p_c^n(\omega_m^n) = \frac{c^n}{(1+c)^{n(n+1)/2}} = \left(\frac{c}{1+c} \right)^n \frac{1}{(1+c)^{n(n-1)/2}}$$

Denoting the “complete” paths by $\omega_a, \omega_m \in \Omega$ we conclude that

$$\nu_c(\omega_a) = \lim_{n \rightarrow \infty} p_c^n(\omega_a^n) = 0$$

$$\nu_c(\omega_m) = \lim_{n \rightarrow \infty} p_c^n(\omega_m^n) = 0$$

It follows that $\{\omega_a\}, \{\omega_m\} \in \mathcal{S}(\rho_n)$ and $\mu(\omega_a) = \mu(\omega_m) = 0$ for any q -propensity in the percolation dynamics. Moreover, not only is $p_c^n(\omega_m^n)$ much smaller than $p_c^n(\omega_a^n)$ for large n but we have

$$\lim_{n \rightarrow \infty} \frac{p_c^n(\omega_m^n)}{p_c^n(\omega_a^n)} = 0$$

These same general results hold for the factorial dynamical $c_k = 1/(k!)$.

As we have seen, there are indications that Ω^{MIX} dominates Ω^{ANT} and Ω^{MAT} . We now examine Ω^{MIX} more closely. We can partition Ω^{MIX} into two parts

$$\begin{aligned}\Omega_{\text{A}}^{\text{MIX}} &= \{\omega = \omega_1\omega_2\cdots \in \Omega^{\text{MIX}} : \omega_2 = x_2\} \\ \Omega_{\text{M}}^{\text{MIX}} &= \{\omega = \omega_1\omega_2\cdots \in \Omega^{\text{MIX}} : \omega_2 = x_3\}\end{aligned}$$

and as usual we define $\Omega_{\text{A},n}^{\text{MIX}}$, $\Omega_{\text{M},n}^{\text{MIX}}$ as the truncated n -paths from $\Omega_{\text{A}}^{\text{MIX}}$, $\Omega_{\text{M}}^{\text{MIX}}$, respectively. Since $\Omega_{\text{A},3}^{\text{MIX}} = \{x_1x_2x_6\}$ and $\Omega_{\text{M},3}^{\text{MIX}} = \{x_1x_3x_6\}$, we have that

$$\begin{aligned}p_c^3(\Omega_{\text{A},3}^{\text{MIX}}) &= \frac{c_0c_1}{(c_0 + c_1)(c_0 + 2c_1 + c_2)} \\ p_c^3(\Omega_{\text{M},3}^{\text{MIX}}) &= \frac{2c_0c_1}{(c_0 + c_1)(c_0 + 2c_1 + c_2)}\end{aligned}$$

Hence, for any value of the coupling constants we have

$$\frac{p_c^3(\Omega_{\text{M},3}^{\text{MIX}})}{p_c^3(\Omega_{\text{A},3}^{\text{MIX}})} = 2$$

Continuing we have that

$$\begin{aligned}\Omega_{\text{A},4}^{\text{MIX}} &= \{x_1x_2x_4x_{14}, x_1x_2x_5x_{15}, x_1x_2x_6x_{14}, x_1x_2x_6x_{15}, \\ &\quad x_1x_2x_6x_{16}, x_1x_2x_6x_{17}, x_1x_2x_6x_{18}, x_1x_2x_6x_{19}\} \\ \Omega_{\text{M},4}^{\text{MIX}} &= \{x_1x_3x_6x_{14}, x_1x_3x_6x_{15}, x_1x_3x_6x_{16}, x_1x_3x_6x_{17}, \\ &\quad x_1x_3x_6x_{18}, x_1x_3x_6x_{19}, x_1x_3x_7x_{18}, x_1x_3x_8x_{19}\}\end{aligned}$$

Hence,

$$\begin{aligned}p_c^4(\Omega_{\text{A},4}^{\text{MIX}}) &= p_c^3(x_1x_2x_6) + p_c^4(x_1x_2x_4x_{14}) + p_c^4(x_1x_2x_5x_{15}) \\ &= \frac{c_0c_1}{\lambda_c(1,0)\lambda_c(2,0)} + \frac{c_0c_1(2c_1 + c_2)}{\lambda_c(1,0)\lambda_c(2,0)\lambda_c(3,0)} \\ p_c^4(\Omega_{\text{M},4}^{\text{MIX}}) &= p_c^3(x_1x_3x_6) + p_c^4(x_1x_3x_7x_{18}) + p_c^4(x_1x_3x_8x_{19}) \\ &= \frac{2c_0c_1}{\lambda_c(1,0)\lambda_c(2,0)} + \frac{c_0c_1(3c_1 + 2c_2)}{\lambda_c(1,0)\lambda_c(2,0)\lambda_c(3,0)}\end{aligned}$$

and the ratio becomes

$$\frac{p_c^4(\Omega_{\text{M},4}^{\text{MIX}})}{p_c^4(\Omega_{\text{A},4}^{\text{MIX}})} = \frac{5c_0 + 6c_1 + 8c_2 + 2c_3}{c_0 + 5c_1 + 4c_2 + c_3}$$

In the percolation dynamics the ratio r_c^4 becomes

$$r_c^4 = \frac{5 + 6c + 8c^2 + 2c^3}{1 + 5c + 4c^2 + c^3}$$

In particular, $r_1^4 = 21/11$ and for $c \geq 1$, $r_c^4 \approx 2$. Moreover, for small c we have $r_c^4 \approx 5$. For the factorial dynamics $c_k = 1/k!$ the ratio becomes $r^4 \approx 1.88$.

We shall not continue this line of thought to find $p_c^5(\Omega_{A,5}^{\text{MIX}})$ and $p_c^5(\Omega_{M,5}^{\text{MIX}})$ because it becomes much more involved. However, we can observe that

$$\begin{aligned}\nu_c(\Omega_A^{\text{MIX}}) &= \lim_{n \rightarrow \infty} p_c^n(\Omega_{A,n}^{\text{MIX}}) \\ \nu_c(\Omega_M^{\text{MIX}}) &= \lim_{n \rightarrow \infty} p_c^n(\Omega_{M,n}^{\text{MIX}})\end{aligned}$$

For any coupling constants, it is clear that

$$p_c^n(\Omega_{M,n}^{\text{MIX}}) > p_c^n(\Omega_{A,n}^{\text{MIX}})$$

so that the ratio

$$r = \frac{\nu_c(\Omega_M^{\text{MIX}})}{\nu_c(\Omega_A^{\text{MIX}})} > 1$$

We conjecture that r is relatively large. Moreover, once we know the DQP ρ_n , $n = 1, 2, \dots$, we conjecture that $\Omega_A^{\text{MIX}}, \Omega_M^{\text{MIX}} \in \mathcal{S}(\rho_n)$ and that r is related to $\mu(\Omega_M^{\text{MIX}})/\mu(\Omega_A^{\text{MIX}})$. Finally this latter ratio may be the ratio of matter to antimatter in our physical universe.

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